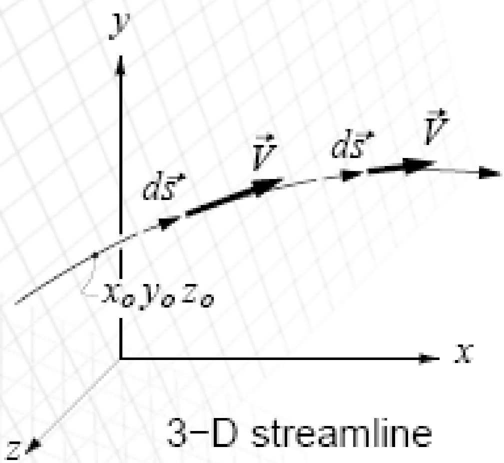
< 2.11. Pathlines, Streamlines, and Streaklines >

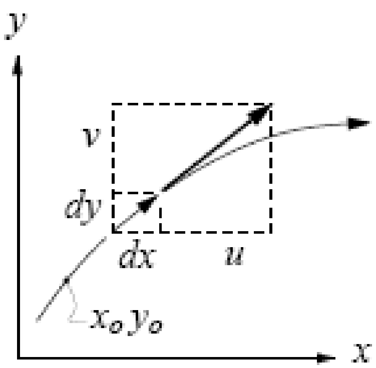
- **Pathline**
  - Path through space as a function of time
  - Same particle (Lagrangian View)
- **Streamline**
  - Parallel to velocity
  - Different particles (Eulerian View)
- **Streakline**
  - Associated with a particular point P in space which has the fluid moving past it
  - Different particle
- **If steady → pathline = streamline = streakline**

< 2.11. Pathlines, Streamlines, and Streaklines >

❖ Streamline



3−D streamline



2−D streamline

$$\vec{ds} \times \vec{v} = 0$$

$$= \begin{vmatrix} i & j & k \\ dx & dy & dz \\ u & v & w \end{vmatrix}$$

In 2D

$$v\,dx - u\,dy = 0$$

$$\frac{dy}{dx} = \frac{v}{u}$$

< 2.12. Angular velocity, Vorticity, and Strain >

❖ **Angular velocity :** $\vec{\omega} = \frac{1}{2} \nabla \times \vec{V}$

❖ **Vorticity :** $\vec{\xi} = \nabla \times \vec{V} = 2\vec{\omega}$

- Rotational flow : $\vec{\xi} \neq 0$
- Irrotational flow : $\vec{\xi} = 0$

$$\vec{\xi} = \nabla \times \vec{V} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

In 2D

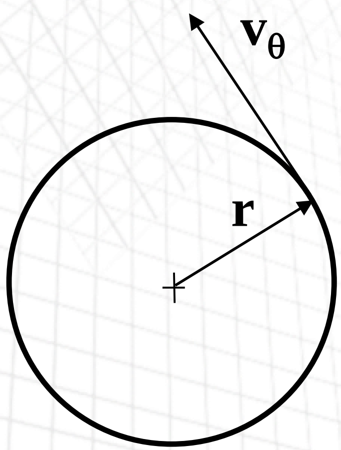
$$\vec{\xi} = \left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$$

< 2.13. Circulation >

❖ Strength of Vorticity (Vortex)

$$\Gamma = -\oint_C \vec{V} \bullet \vec{ds} = -\iint_S (\nabla \times \vec{V}) \bullet \vec{dS} = -\iint_S \vec{\xi} \bullet \vec{dS}$$

Stokes Theorem



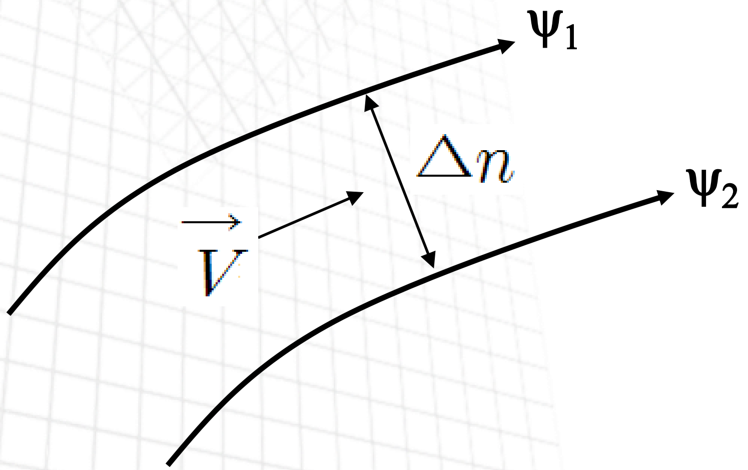
$$\Gamma = v_\theta 2\pi r \Rightarrow v_\theta = \frac{\Gamma}{2\pi r}$$

< 2.14. Stream Function >

- In 2D, streamline can be expressed as

$$u\,dy - v\,dx = 0$$

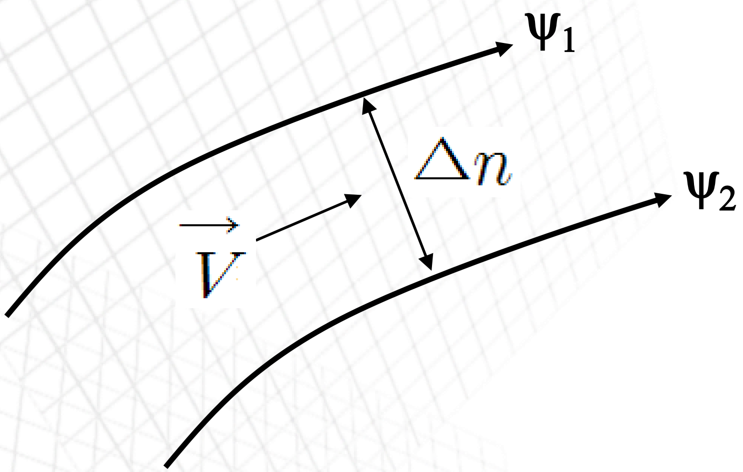
- Let's define a function which satisfies the above equation, stream function $\psi(x,y) = c$



$$\Delta\psi = \vec{V}\Delta n$$

$$\vec{V} = \lim_{\Delta n \to 0} \frac{\Delta\psi}{\Delta n} = \frac{\partial\psi}{\partial n}$$

< 2.14. Stream Function >



$$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy = 0$$
$$= -v\,dx + u\,dy$$

$$\therefore \frac{\partial \psi}{\partial x} = -v\,, \quad \frac{\partial \psi}{\partial y} = u$$

➔

In polar coordinate

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

$$v_\theta = -\frac{\partial \psi}{\partial r}$$

< 2.15. Velocity Potential >

- **Vorticity :** $\vec{\xi} = \nabla \times \overrightarrow{V}$ **Irrotational :** $\nabla \times \overrightarrow{V} = 0$

- **Using vector identity** $\nabla \times (\nabla \phi) = 0$

$$\overrightarrow{V} = \nabla \phi \Rightarrow \phi$$

: Velocity potential

$$\nabla \times \overrightarrow{V} = \nabla \times (\nabla \phi) = 0$$

< 2.15. Velocity Potential >

- $\overrightarrow{V} = \nabla\phi$

$$\vec{v} = u_i + v_j + w_k = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$$

$$u = \frac{\partial\phi}{\partial x},\ v = \frac{\partial\phi}{\partial y},\ w = \frac{\partial\phi}{\partial z}$$

In cylindrical coordinate, $v_r = \frac{\partial\phi}{\partial r},\ v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta},\ v_z = \frac{\partial\phi}{\partial z}$

In spherical coordinate, $v_r = \frac{\partial\phi}{\partial r},\ v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta},\ v_\phi = \frac{1}{r sin\theta}\frac{\partial\phi}{\partial\Phi}$

< 2.15. Velocity Potential >

❖ Velocity potential is differentiated parallel to the velocity whereas the stream function is differentiated normal to the velocity.

$$d\psi = \underset{\substack{\Uparrow \\ -v}}{\frac{\partial \psi}{\partial x}} dx + \underset{\substack{\Uparrow \\ u}}{\frac{\partial \psi}{\partial y}} dy = 0 \quad \rightarrow \quad \left(\frac{dy}{dx}\right)_{\psi = const} = \frac{v}{u}$$

$$d\phi = \underset{\substack{\Uparrow \\ u}}{\frac{\partial \phi}{\partial x}} dx + \underset{\substack{\Uparrow \\ v}}{\frac{\partial \phi}{\partial y}} dy = 0 \quad \rightarrow \quad \left(\frac{dy}{dx}\right)_{\phi = const} = -\frac{u}{v}$$

$$\therefore \left(\frac{dy}{dx}\right)_{\psi = const} \bullet \left(\frac{dy}{dx}\right)_{\phi = const} = -1$$

< 2.15. Velocity Potential >

- **Velocity potential** is defined for **irrotational flow** only whereas the stream function is used for both irrotational & rotational flow.

- Velocity potential applied to 3D, whereas stream function is defined for 2D only.